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Time and position for occurrence of relativistic nucleon-nucleus scattering in the Dirac approach

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Abstract. As a temporal approach a method of examining occurrence of nuclear collisions is exploited in terms of the wave packet description. The off-shell scattering matrix element characteristic of the method governs the time and position when reactions take place during interactions. For a plain exemplification we consider occurrence of relativistic nucleon-nucleus scattering in the Dirac phenomenology. The results indicate that at higher intermediate energies the forward scattering occurs before the peak of the nucleon probability density reaches the center of the nuclear interaction region.

PACS. 03.65.Pm Relativistic wave equations – 11.80.Fv Approximations (eikonal approximation, variational principles, etc.) – 25.40.Cm Elastic proton scattering

1 Introduction

The conception of time and position when nuclear collisions occur during interactions will develop novel versions of reaction mechanisms found particularly at intermediate and high energies. The time and position indicate for instance when and where the scattering of a particle takes place for a given angle together with the generation of some excited state of the target in the course of the interaction. Furthermore, these would serve to examine features of particle production and/or emission in more complicated collisions. Let the time and the position be hereafter designated as occurrence time and occurrence position, respectively.

In a previous paper [1] the occurrence time is defined in terms of the wave packet which emerges due to the interaction. It thereby turns out that even for a reaction concerned with sufficiently small wavelength the time found by this method deviates considerably from that of a conventional classical picture using trajectories for scattering. This suggests further exploitation of the quantum formalism for the occurrence of collisions.

Reactions with sufficiently short wavelength have often been discussed by means of semiclassical and classical approaches. As for occurrence of scattering, however, primitive methods of using trajectories do not always yield reasonable results since they disregard significant quantum features, and have difficulty in handling, for example, effects of nuclear absorption. There exist different types of nuclear reaction with small wavelength which have characteristics beyond classical versions. In scattering described by the Dirac spinor such as that in the Dirac phenomenology [2], for example, contribution of the lower component in the spinor increases with increasing incident energy, which has no classical analog and is expected to cause some peculiar behavior of the wave packet.

In this paper consideration of occurrence of relativistic nucleon-nucleus elastic scattering during the interaction is performed in terms of the wave packet description of the Dirac phenomenology as a plain exemplification of occurrence of nuclear scattering with sufficiently small wavelength. While the occurrence time alone is considered in [1] we here discuss the occurrence position also. The off-shell scattering matrix element defined in the present approach is to determine the time as before, and here the time is employed to find the position using the nucleon probability density. This procedure indicates how to deal with occurrence of quantum collisions, and leads to the demonstration that even for the primitive and basic scattering mentioned above the mechanism of the occurrence is beyond classical versions due to some features of the off-shell matrix element and despite the short wavelength. Discussion of the occurrence is made also in reference to some novelty of the wave packet behavior due to characteristics of the Dirac approach at higher intermediate energies [3].

In sect. 2 the occurrence time formula is derived in a different manner from the previous one. In sect. 3 the occurrence time and the incident nucleon probability density are represented in the eikonal approach in order to define and discuss the occurrence position. Section 4 describes results and discussion. Conclusion is given in sect. 5.

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2 Occurrence time

The occurrence time provides a temporal description of quantum collisions and differs from the time delay [4] in significance and consequent form. As performed in [1] derivation of its formula is made here on the basis of the wave packet emerging during the interaction. Although the formula is identical with the previous one the present derivation is straightforward.

Suppose that the relevant Dirac equations read

$$\big[H_0 + W - E(k)\big]\psi^{(+)}(\mathbf{k},\mathbf{r}) = 0, \tag{2.1}$$

$$\big[H_0 - E(p)\big]\phi(\mathbf{p},\mathbf{r}) = 0, \tag{2.2}$$

where $E(k) = \sqrt{k^2+m^2}$, $E(p) = \sqrt{p^2+m^2}$, and

$$H_0 = \boldsymbol{\alpha}\cdot(-i\nabla) + \beta m, \qquad W = V(r) + \beta S(r), \tag{2.3}$$

V and S being the Lorentz vector and scalar potentials. And furthermore, $\mathbf{k}$ and $\mathbf{p}$ are the initial and the final wave vectors. Assuming that the spin component is not changed during the interaction, we omit the index. Then the wave packet representation of the full spinor takes the form

$$\psi(\mathbf{k},\mathbf{r},t) = \int \frac{\mathrm{d}^3 q}{E(k')} a(\mathbf{q}) e^{-iE(k')(t-t_0)} \psi^{(+)}(\mathbf{k}',\mathbf{r}), \tag{2.4}$$

where $\mathbf{q} = \mathbf{k}' - \mathbf{k}$ and the amplitude $a(\mathbf{q})$ is taken to be real so that the center of the free wave packet with $W = 0$ would pass the origin $\mathbf{r} = 0$ at $t = t_0$.

When an incident wave packet reaches the interaction region, different wave packets corresponding to each of the channels arise due to the interaction. As mentioned previously, the definition of the occurrence time is concerned with the emerging wave packets written as

$$X(\mathbf{k},t) = \int \frac{\mathrm{d}^3 q}{E(k')} a(\mathbf{q}) \frac{e^{-iE(k')(t-t_0)}}{E^{(+)}(k') - H_0} W\psi^{(+)}(\mathbf{k}'). \tag{2.5}$$

Then we consider a measure of the overlap of the final state $\phi(\mathbf{p})$ with the arising wave packet:

$$\Omega(\mathbf{p},\mathbf{k},t) = |\langle\phi(\mathbf{p}) \mid X(\mathbf{k},t)\rangle|, \tag{2.6}$$

where $\mathbf{p}$ is, for the time being, taken arbitrarily and is then independent of $\mathbf{k}$ in the sense that X is associated with different wave vectors. While $\Omega = 0$ for $t = -\infty$, the magnitude of Ω increases with the emergence of the wave packet, and approaches some nonvanishing constant as $t \to \infty$. Suppose that the scattering occurs when Ω amounts to some magnitude. That is, the occurrence time is presumed to be the time corresponding to the peak which the time derivative $\partial\Omega/\partial t$ has as a function of time for given $\mathbf{p}$ and $\mathbf{k}$.

From (2.2) and (2.5) it follows that:

$$\big\langle\phi(\mathbf{p}) \mid X(\mathbf{k},t)\big\rangle = \int \frac{\mathrm{d}^3 q}{E(k')} a(\mathbf{q}) \times \frac{e^{-iE(k')(t-t_0)}}{E^{(+)}(k') - E(p)} T(\mathbf{p},\mathbf{k}'), \tag{2.7}$$

where $T(\mathbf{p},\mathbf{k}')$ is the off-shell matrix element represented as

$$T(\mathbf{p},\mathbf{k}') = \big\langle\phi(\mathbf{p})|W|\psi^{(+)}(\mathbf{k}')\big\rangle = |T(\mathbf{p},\mathbf{k}')|e^{i\eta(\mathbf{p},\mathbf{k}')}. \tag{2.8}$$

Defining

$$L(\mathbf{p},\mathbf{k},\lambda) = \int \frac{\mathrm{d}^3 q}{E(k')} a(\mathbf{q}) T(\mathbf{p},\mathbf{k}') e^{-i\Delta E(\lambda - t_0)}, \tag{2.9}$$

and using

$$\frac{1}{\omega'} = -i\int_{-\infty}^{t} \mathrm{d}\lambda\, e^{i\omega'(t-\lambda)},$$

where $\Delta E = E(k') - E(k)$ and $\omega' = E^{(+)}(k') - E(p)$, expression (2.7) is then rewritten as

$$\big\langle\phi(\mathbf{p})|X(\mathbf{k},t)\big\rangle = -ie^{-iE(k)(t-t_0)}\int_{-\infty}^{t} \mathrm{d}\lambda e^{i\omega(t-\lambda)} L(\mathbf{p},\mathbf{k},\lambda), \tag{2.10}$$

where $\omega = E^{(+)}(k) - E(p)$. We assume that in (2.8) $|T(\mathbf{p},\mathbf{k}')| \simeq |T(\mathbf{p},\mathbf{k})|$ and $\eta(\mathbf{p},\mathbf{k}') = \eta(\mathbf{p},\mathbf{k}) + \mathbf{q}\cdot\nabla_k\eta(\mathbf{p},\mathbf{k})$ with $\mathbf{p}$ independent of $\mathbf{k}$, and in (2.9) $\Delta E = \mathbf{q}\cdot\mathbf{u}$, where ∇_k is the gradient with respect to $\mathbf{k}$ and $\mathbf{u} = \nabla_k E(k) = \mathbf{k}/E(k)$ is the incident velocity. Then, by means of

$$G(\mathbf{X}) = \int \frac{\mathrm{d}^3 q}{E(k')} a(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{X}}, \tag{2.11}$$

there results

$$L(\mathbf{p},\mathbf{k},\lambda) = T(\mathbf{p},\mathbf{k}) G\big[\nabla_k\eta(\mathbf{p},\mathbf{k}) - \mathbf{u}(\lambda - t_0)\big]. \tag{2.12}$$

Now let the amplitude be taken as

$$a(\mathbf{q}) = Ae^{-d^2q^2}, \tag{2.13}$$

A and $d(>0)$ being constant. We thereby find

$$G(\mathbf{X}) = B\exp\left[-\frac{\mathbf{X}^2}{(2d)^2}\right], \qquad B = \frac{A}{E(k)}\left(\frac{\sqrt{\pi}}{d}\right)^3, \tag{2.14}$$

where $E(k')$ in (2.11) is replaced by $E(k)$. On writing the phase as $\eta(\mathbf{p},\mathbf{k}) = \eta(p,k,\theta)$, where θ is the angle between $\mathbf{k}$ and $\mathbf{p}$, and using

$$\nabla_k\eta = \left(\frac{\partial\eta}{\partial k} + \frac{\cot\theta}{k}\frac{\partial\eta}{\partial\theta}\right)\frac{\mathbf{k}}{k} - \frac{1}{k\sin\theta}\left(\frac{\partial\eta}{\partial\theta}\right)\frac{\mathbf{p}}{p},$$

G in (2.12) is found to be

$$G = C(p,k,\theta)e^{-\{w(\lambda)\}^2}, \tag{2.15}$$

where

$$w(\lambda) = \frac{u}{2d}\left(\lambda - t_0 - \frac{1}{u}\frac{\partial\eta}{\partial k}\right). \tag{2.16}$$

Then it follows that

$$\Omega(\mathbf{p},\mathbf{k},t) = \big|C(p,k,\theta)T(\mathbf{p},\mathbf{k})\big|\left|\int_{-\infty}^{t}\mathrm{d}\lambda\, e^{i\omega(t-\lambda)}e^{-\{w(\lambda)\}^2}\right|. \quad (2.17)$$

Until now the derivative $\nabla_k\eta$ or $\partial\eta/\partial k$ is given by regarding $\mathbf{p}$ or p as being independent of $\mathbf{k}$ or k. Here we set $p = k$, and thus $\omega = 0$ in (2.17). Now in accordance with the above-mentioned definition the occurrence time corresponds to the maximum in the time derivative of (2.17) with $p = k$, and turns out to take the form

$$\tau(k,\theta) = t_0 + \frac{1}{u}\left(\frac{\partial}{\partial k}\eta(p,k,\theta)\right)_{p=k}. \quad (2.18)$$

The derivative is hence defined by differentiating the off-shell phase $\eta(p,k,\theta)$ with respect to the incident wave number k first and then putting $p = k$.

The time delay is given from the on-shell matrix element [4], and is shown to be connected with the time (2.18) as follows. Let the on-shell matrix element be obtained from $T(\mathbf{p},\mathbf{k})$ by putting $p = k$, and be written as

$$\overline{T}(\mathbf{p},\mathbf{k}) = \overline{T}(k,\theta) = |\overline{T}|e^{i\overline{\eta}}. \quad (2.19)$$

Then using $\overline{\eta}(k,\theta) = \eta(p = k, k, \theta)$, we here write the time delay in the form

$$\overline{\tau} = \frac{1}{u}\frac{\partial\overline{\eta}}{\partial k} = \frac{1}{u}\left[\left(\frac{\partial\eta}{\partial p}\right)_{p=k} + \left(\frac{\partial\eta}{\partial k}\right)_{p=k}\right], \quad (2.20)$$

which provides

$$\tau = t_0 + \overline{\tau} - \frac{1}{u}\left(\frac{\partial\eta}{\partial p}\right)_{p=k}, \quad (2.21)$$

where $(\partial\eta/\partial p)_{p=k}$ is given by setting $p = k$ in $\partial\eta/\partial p$ with p independent of k. The time delay is quite small and then the last term will be the major one. We adopt this form for our computation and discussion. Further, the derivatives of the phases are represented as

$$\frac{\partial\overline{\eta}}{\partial k} = \mathrm{Im}\left(\frac{\partial\overline{T}}{\partial k}/\overline{T}\right), \qquad \frac{\partial\eta}{\partial\xi} = \mathrm{Im}\left(\frac{\partial T}{\partial\xi}/T\right) \quad (2.22)$$

with $\xi = p,\ k$, which will also be used later.

The occurrence time in our approach is defined in terms of the emerging wave packet or (2.6) and thus has no classical analog in contrast to the time delay for nonrelativistic scattering with small wavelength which occasionally gives a picture associated with classical trajectories as shown in the WKB method.

3 Occurrence time and position in the eikonal approach

The occurrence time is designed to determine the occurrence position in terms of the nucleon probability density given by the wave packet. In order to define and find the occurrence position by analytic evaluations we employ the eikonal approach, which is first outlined for our procedure.

In the eikonal approximation [5] the full spinor takes the form

$$\psi^{(+)}(\mathbf{k},\mathbf{r}) = N(k)\begin{pmatrix} e^{i\{\mathbf{k}\cdot\mathbf{r}+Q(\mathbf{k},\mathbf{r})\}}\chi \\ -\frac{i}{D(k,r)}\boldsymbol{\sigma}\cdot\nabla e^{i\{\mathbf{k}\cdot\mathbf{r}+Q(\mathbf{k},\mathbf{r})\}}\chi\end{pmatrix},$$

$$N(k) = \sqrt{\frac{E(k)+m}{2m(2\pi)^3}}, \quad (3.1)$$

where χ is the two component spinor and $D(k,r) = E(k) + m + S(r) - V(r)$. Further we have

$$Q(\mathbf{k},\mathbf{r}) = Q_0(k,z,b) + Q_1(k,z,b)\boldsymbol{\sigma}\cdot(\mathbf{b}\times\mathbf{k}), \quad (3.2)$$

$$Q_0 = -\frac{m}{k}\int_{-\infty}^{z}\mathrm{d}z'\big[U_{\mathrm{C}}(k,r') - ikz'U_{\mathrm{SO}}(k,r')\big], \quad (3.3)$$

$$Q_1 = -\frac{m}{k}\int_{-\infty}^{z}\mathrm{d}z'\, U_{\mathrm{SO}}(k,r'), \quad (3.4)$$

$$U_{\mathrm{C}}(k,r) = S(r) + \frac{E(k)}{m}V(r) + \frac{S(r)^2 - V(r)^2}{2m}, \quad (3.5)$$

$$U_{\mathrm{SO}}(k,r) = \frac{1}{2mD(k,r)}\frac{1}{r}\frac{\partial}{\partial r}\{V(r) - S(r)\}, \quad (3.6)$$

where $\mathbf{r} = \mathbf{b} + z\mathbf{n}$, $\mathbf{n} = \mathbf{k}/k$, $\mathbf{b}\cdot\mathbf{k} = 0$, and $r' = \sqrt{z'^2 + b^2}$.

3.1 The off-shell matrix element and the occurrence time

In order to present the occurrence time in the eikonal approach, the off-shell matrix element (2.8) is written in the eikonal form using (3.1) and the free solution

$$\phi(\mathbf{p},\mathbf{r}) = N(p)\begin{pmatrix} e^{i\mathbf{p}\cdot\mathbf{r}}\chi \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}e^{i\mathbf{p}\cdot\mathbf{r}}\chi}{E(p)+m}\end{pmatrix},$$

$$N(p) = \sqrt{\frac{E(p)+m}{2m(2\pi)^3}}. \quad (3.7)$$

From manipulations given in Appendix A we first have

$$T(\mathbf{p},\mathbf{k}) = N(p)N(k)\chi^\dagger\hat{T}(\mathbf{p},\mathbf{k})\chi, \quad (3.8)$$

where $\mathbf{k}$ with $p \neq k$ is for brevity employed instead of $\mathbf{k}'$ in (2.8), and

$$\hat{T}(\mathbf{p},\mathbf{k}) = 2m\int\mathrm{d}^3x\frac{e^{-i\mathbf{s}\cdot\mathbf{r}}}{D} \times\big[\hat{U}_{\mathrm{C}} + \hat{U}_{\mathrm{SO}}\{i\mathbf{p}\cdot\mathbf{r} + \boldsymbol{\sigma}\cdot(\mathbf{r}\times\mathbf{p})\}\big]e^{iQ}, \quad (3.9)$$

with $\mathbf{s} = \mathbf{p} - \mathbf{k}$ and

$$\hat{U}_{\mathrm{C}}(p,k,r) = U_{\mathrm{C}}(k,r) + \frac{E(k)-E(p)}{2m}\big\{S(r) - V(r)\big\}, \quad (3.10)$$

$$\hat{U}_{\mathrm{SO}}(p,k,r) = \frac{E(k)+m}{E(p)+m}U_{\mathrm{SO}}(k,r). \quad (3.11)$$

On writing $\chi^\dagger \hat{T}(\mathbf{p},\mathbf{k})\chi = F^{(+)}(p,k,\theta)+F^{(-)}(p,k,\theta)$, and referring again to Appendix A, it follows that

$$F^{(\pm)}(p,k,\theta)=2\pi m\int_0^\infty \mathrm{d}b\, b\int_{-\infty}^{\infty}\mathrm{d}z\frac{e^{i[(k-p\cos\theta)z+Q_\pm]}}{D(k,r)} \times \big[J_0(pb\sin\theta)\hat{U}_{\mathrm{C}}(p,k,r) + \{J_0(pb\sin\theta)\cos\theta \pm J_1(pb\sin\theta)\sin\theta\} \times \hat{U}_{\mathrm{SO}}(p,k,r)p(iz\pm b)\big], \tag{3.12}$$

where

$$Q_\pm = Q_0 \pm kbQ_1. \tag{3.13}$$

Hereafter, we consider the forward scattering. Furthermore, the Coulomb interaction in the nuclear region is minor compared to the vector potential, and is then disregarded. Let $T(p,k)$ denote the off-shell matrix element (3.8) for $\theta=0$:

$$T(p,k) = N(p)N(k)\big[F^{(+)}(p,k)+F^{(-)}(p,k)\big], \tag{3.14}$$

where

$$F^{(\pm)}(p,k) = F^{(\pm)}(p,k,\theta=0) = 2\pi m\int_0^\infty \mathrm{d}b\ b\int_{-\infty}^{\infty}\mathrm{d}z\, M_\pm(p,k,z,b), \tag{3.15}$$

with

$$M_\pm = \frac{e^{i[(k-p)z+Q_\pm]}}{D}\big[\hat{U}_{\mathrm{C}}+\hat{U}_{\mathrm{SO}}p(iz\pm b)\big]. \tag{3.16}$$

Bearing in mind

$$M_\pm(p=k,k,z,b) = \frac{ik}{m}\frac{\partial}{\partial z}\left(\frac{e^{iQ_\pm}}{D}\right),$$

we have the on-shell matrix element

$$\overline{T}(k) = T(p=k,k) = \big(N(k)\big)^2\frac{4\pi ik}{E(k)+m}g(k), \tag{3.17}$$

with

$$g(k) = \int_0^\infty \mathrm{d}b\ b\left[\frac{1}{2}\big(e^{i\chi_+}+e^{i\chi_-}\big)-1\right], \tag{3.18}$$

where $\chi_\pm = \chi_{\mathrm{C}} \pm \chi_{\mathrm{SO}}$ and

$$\chi_{\mathrm{C}} = -\frac{m}{k}\int_{-\infty}^{\infty}\mathrm{d}z\, U_{\mathrm{C}}(k,r),$$

$$\chi_{\mathrm{SO}} = -\frac{m}{k}(kb)\int_{-\infty}^{\infty}\mathrm{d}z\, U_{\mathrm{SO}}(k,r). \tag{3.19}$$

As made before the derivative in (2.18) is rewritten as $(\partial_k\eta)_{p=k} = \partial_k\overline{\eta} - (\partial_p\eta)_{p=k}$, where $\partial_\xi = \partial/\partial\xi$. Then using (2.22) each term in the right-hand side becomes

$$\partial_k\overline{\eta} = \mathrm{Im}\left(\frac{\partial_k g(k)}{g(k)}\right), \tag{3.20}$$

$$\big(\partial_p\eta\big)_{p=k} = -\,\mathrm{Re}\left(\frac{J(k)}{g(k)}\right), \tag{3.21}$$

where

$$J(k) = \frac{m}{2k}\big(E(k)+m\big)\int_0^\infty \mathrm{d}b\, b\int_{-\infty}^{\infty}\mathrm{d}z\big\{\partial_p(M_+ + M_-)\big\}_{p=k}, \tag{3.22}$$

with

$$\big(\partial_p M_\pm\big)_{p=k} = \frac{e^{iQ\pm}}{D}\left[-izU_{\mathrm{C}} + \frac{k}{2mE(k)}(V-S) + (iz\pm b)U_{\mathrm{SO}}\left(\frac{m}{E(k)}-ikz\right)\right]. \tag{3.23}$$

Since the potentials S and V vary comparatively slowly with respect to the wave number k we assume $\partial_k S = \partial_k V = 0$ in the derivative $(\partial_k\eta)_{p=k}$ or $\partial_k\overline{\eta}$. On the other hand, the explicit energy dependence due to $E(k)$ in (3.5) gives rise to some characteristics for the scattering at intermediate energies as a result of the appreciable contribution of the lower component of the Dirac spinor, which is to be described in sect. 4. Note that the first term in square brackets in (3.23) results from differentiation with respect to p of the function $e^{i(k-p)z}$ in (3.16) peculiar to the off-shell matrix element. The term dominantly contributes to the integral (3.22), and is crucial to the occurrence, which will also be discussed in sect. 4

3.2 The probability density and the occurrence position

The occurrence position is assumed to be the point corresponding to the maximum of the nucleon probability density at the occurrence time. In terms of (2.4) the probability density is represented as

$$\rho(\mathbf{k},\mathbf{r},t) = \psi^\dagger(\mathbf{k},\mathbf{r},t)\psi(\mathbf{k},\mathbf{r},t). \tag{3.24}$$

Using (3.1) this becomes

$$\rho(k,z,b,t) = \frac{I}{2}\big[e^{-\Phi_+}+e^{-\Phi_-}\big], \tag{3.25}$$

with $I = B^2\Lambda$ and

$$\Phi_\pm(k,z,b,t) = \frac{1}{2d^2}\Big[b^2+\big\{z-u\big(t-t_0\big)+\mathrm{Re}\,\partial_k Q_\pm\big\}^2 - \big(\mathrm{Im}\,\partial_k Q_\pm\big)^2\Big] + 2\,\mathrm{Im}\,Q_\pm, \tag{3.26}$$

the derivation of which is given in Appendix B.

The peak moves along the line with $b=0$, and then in view of (3.13) we set $Q_\pm = Q_0(k,z,b=0)$. Let $z=\zeta$ be the position of the peak at the occurrence time. Writing $\Phi(k,z,t) = \Phi_+(k,z,b=0,t) = \Phi_-(k,z,b=0,t)$ and employing (2.18) with $\theta=0$, the occurrence position ζ is, therefore, obtained by finding the point corresponding to the minimum of the form

$$\Phi(k,z,t=\tau) = \frac{1}{2d^2}\Big[\big\{z-\big(\partial_k\eta\big)_{p=k}+\mathrm{Re}\,\partial_k Q_0\big\}^2 - \big(\mathrm{Im}\,\partial_k Q_0\big)^2\Big] + 2\,\mathrm{Im}\,Q_0. \tag{3.27}$$

Table 1. The potential parameters for (4.1) are from [5] for p-^{40}Ca at $T_p = 500\,\mathrm{MeV}$ and from [6] for p-^{208}Pb at $T_p = 800\,\mathrm{MeV}$, and the values of τ are given with choice $t_0 = 0$ and those of ζ for $d = 1.0\,\mathrm{fm}$.

T_p (MeV)	S_0 (MeV)	V_0 (MeV)	R (fm)	a(fm)	τ (10^{-24} s)	ζ (fm)
500	$-303 + i73$	$191 - i86$	3.55	0.64	-4.0	-1.2
800	$-273 + i97$	$154 - i109$	6.60	0.63	-8.2	-2.6

Here we assume that I in (3.25) is irrelevant to the determination of ζ because of $\Lambda(k,r) \simeq N(k)^2$. Obviously, the position depends on the wave packet width d on account of nuclear absorption.

4 Results and discussion

In our wave packet formalism the occurrence time is defined, and the occurrence position is thereby obtained. The time is given independently of the wave packet width whereas the position depends on the width. In the following we discuss the occurrence of the scattering in connection with some features of the wave packet for the Dirac approach.

We employ the potentials in [5,6]:

$$S(r) = S_0 f(r), \qquad V(r) = V_0 f(r),$$
$$f(r) = \left[1 + e^{(r-R)/a}\right]^{-1}. \tag{4.1}$$

The parameters and relevant results are presented in table 1. The terms concerned with U_{SO} are thereby found to be minor for discussion and are disregarded hereafter. We first mention features of the derivatives with respect to the incident wave number generated by expansion in powers of $\mathbf{q}$ in the wave packet formulation. The explicit energy dependence of U_{C} in (3.5) leads to some peculiarity of the derivatives at higher intermediate energies which may not be found for scattering at lower energies. As shown in the Dirac phenomenology the magnitude of $\mathrm{Im}\, U_{\mathrm{C}}$ is greater than that of the repulsive $\mathrm{Re}\, U_{\mathrm{C}}$. However, for the derivative

$$\partial_k Q_0 = \frac{m}{k^2}\int_{-\infty}^{z} \mathrm{d}z' \left(S + \frac{m}{E(k)}V + \frac{S^2 - V^2}{2m}\right), \tag{4.2}$$

where S and V are assumed to be independent of k, as mentioned before, we find

$$-\mathrm{Re}\,\partial_k Q_0 \gg |\mathrm{Im}\,\partial_k Q_0|, \tag{4.3}$$

because of the factor $m/E(k)$, which is a consequence of the appreciable contribution of the lower component. As an example, the scattering at the kinetic energy $T_p = 800\,\mathrm{MeV}$ in table 1 gives

$$\begin{aligned} Q_0 &= \int_{-\infty}^{z} \mathrm{d}z' \big[-\big(4.19f + 9.22f^2\big) \\ &\quad + i\big(34.30f + 3.29f^2\big)\big] \times 10^{-2}, \\ \partial_k Q_0 &= \int_{-\infty}^{z} \mathrm{d}z' \big[\big(-8.32f + 1.24f^2\big) \\ &\quad + i\big(1.67f - 0.45f^2\big)\big] \times 10^{-2}\,\mathrm{fm}. \end{aligned} \tag{4.4}$$

Hence $\mathrm{Im}\,\partial_k Q_0$ in (3.27) does not have much effect on the shape of the probability density and is ignored whereas $\mathrm{Im}\, Q_0$ is crucial to the wave packet behavior.

Now expansion of $E(k')$ made previously yields the terms quadratic in $\mathbf{q}$ as well, which are omitted in our procedure. However, they give rise to the time dependence of the packet width. Nevertheless, instead of a macroscopic wave packet, we may adopt a narrow wave packet the width of which is smaller than the nuclear dimension since on account of high speed the width does not appreciably vary during passage through the interaction region. The time-dependent width leads to the breadth of the probability density which contains the term depending on $|t - t_0|$ as shown in Appendix C. Suppose that the initial wave packet lies at $t = t_i$ near an entrance to the interaction region, *i.e.*, $z = -R$ with $b = 0$ where the packet is almost identical with the free one since the interaction is weak, and the backward scattering amplitude is negligible, which makes the eikonal approximation available therein. Then the density breadth decreases during the interval $t_i < t < t_0$, attains to the minimum at $t \simeq t_0$, and increases for $t > t_0$. Further we have $t_i < \tau < t_0$, and the small difference $t_0 - t_i \simeq R/u(\sim 10^{-23}\mathrm{s})$ due to the high velocity keeps the time-dependent term minor compared to the size d. Accordingly, the shrinkage and the high speed enable one to consider the scattering process using the narrow packet with constant width d to find the occurrence position.

In order to discuss the results the probability density at the occurrence time is, from (3.27), rewritten as

$$\rho(k, z, b = 0, t = \tau) = I e^{-\Phi} = I\rho_1\rho_2, \tag{4.5}$$

where the decomposition is made in such a way that

$$\begin{aligned} \rho_1 &= e^{-\Phi_1}, \qquad \Phi_1 = \frac{1}{2d^2}\left[z - \big(\partial_k \eta\big)_{p=k} + \mathrm{Re}\,\partial_k Q_0\right]^2, \\ \rho_2 &= e^{-\Phi_2}, \qquad \Phi_2 = 2\,\mathrm{Im}\, Q_0, \end{aligned} \tag{4.6}$$

$\mathrm{Im}\,\partial_k Q_0$ being neglected. In fig. 1 the densities are presented in the form of $e^{-\Phi}$ without the minor quantity I. Since $\mathrm{Re}\,\partial_k Q_0 < 0$ the peak of ρ_1 is, in spite of the repulsion $\mathrm{Re}\, U_{\mathrm{C}} > 0$, somewhat ahead of that of the free particle probability density which would pass the point $z = (\partial_k \eta)_{p=k}$ at $t = \tau$. However, decrease of ρ_2 shifts the peak of $e^{-\Phi}$ backward. When the size d is taken greater the shape of the density is distorted still more, and the position $z = \zeta$ is shifted further from the center of the interaction region.

The present results suggested that the scattering occurs before the peak reaches the center. Apart from the

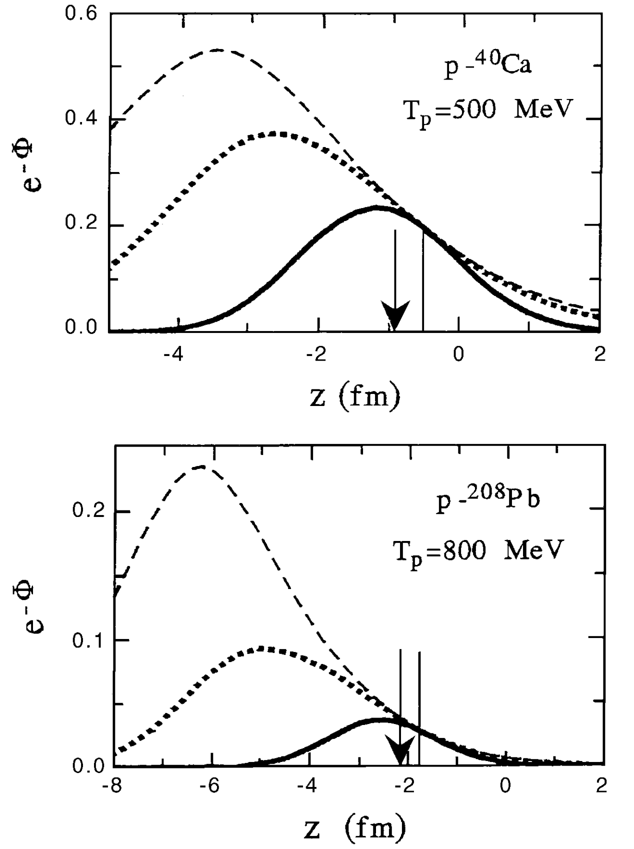

Fig. 1. Probability densities at the occurrence time expressed as $e^{-\Phi}$. The solid, dotted, and dashed curves correspond to $d = 1.0$, 2.0, and 3.0 fm, respectively. The arrow indicates the point $z = (\partial_k\eta)_{p=k}$, *i.e.*, the position of the peak of the probability density for free motion. The bar represents the position corresponding to the maximum of ρ_1 given in (4.6).

effect due to ρ_2 mentioned above this substantially originates, as a numerical result, from

$$-R < \left(\partial_k\eta\right)_{p=k} - \mathrm{Re}\,\partial_k Q_0 = \partial_k\overline{\eta} - \left(\partial_p\eta\right)_{p=k} - \mathrm{Re}\,\partial_k Q_0 < 0, \quad (4.7)$$

which holds independently of z since $\mathrm{Re}\,\partial_k Q_0$ changes slowly. Then the peak of ρ_1 is situated at some point in the region $-\mathrm{Re} < z < 0$. The result (4.7) is due to the dominant term $(\partial_p\eta)_{p=k}(> 0)$, which is qualitatively discussed as follows. The major contribution to the integral (3.22) comes from the first term in square brackets of (3.23) as stated before, and is from the region $-R < z < 0$ with $b \geq 0$ because of the rapid decrease of $e^{-\mathrm{Im}\,Q_0}$ with increasing z. Disregarding minor terms and quantities, the real part of (3.23) becomes

$$\mathrm{Re}\left[\frac{e^{iQ_0}}{D}\big(-izU_{\mathrm{C}}\big)\right] \simeq \frac{e^{-\mathrm{Im}\,Q_0}\cos\big(\mathrm{Re}\,Q_0\big)}{E(k)+m} z\,\mathrm{Im}\,U_{\mathrm{C}} > 0, \quad (4.8)$$

where $\cos(\mathrm{Re}\,Q_0) > 0$ for $z < 0$. On the other hand, we have

$$-\mathrm{Re}\,g(k) \gg |\,\mathrm{Im}\,g(k)|. \quad (4.9)$$

Then we find that $(\partial_p\eta)_{p=k}$ is positive due to (3.21) and major because of the strong absorption, which leads to $-R < \zeta < 0$. For weaker absorption we would have another result since $e^{-\mathrm{Im}\,Q_0}$ slowly decreases and the region of the contribution is then extended.

Now the narrower wave packet will be preferable for determination of the occurrence position as long as the time dependence of the width is negligible. As the packet width d becomes smaller the point $z = \zeta$ approaches the position of the peak of ρ_1 or that of the maximum of the free probability density. According to computation in Appendix C the time dependence can be disregarded for $d = 1.0$ fm. As illustrated in fig. 1, the distance between the two positions stated above is appreciably short compared with the nuclear radius, and the point $z = \zeta$ found by the above choice is sufficiently close to the positions. Therefore, the scattering may be regarded as occurring in the vicinity of them. Then if an approximation $\zeta \simeq (\partial_k\eta)_{p=k}$ is made the formula (2.18) for $\theta = 0$ leads to a direct connection between the occurrence time and position.

The time delay is found to be positive. This does not come out of the repulsion $\mathrm{Re}\,U_{\mathrm{C}} > 0$. Bearing in mind $\chi_{\mathrm{C}}(k,b) = Q_0(k, z=\infty, b)$ and (4.3), it follows from (3.18) that, neglecting minor terms,

$$\mathrm{Im}\,\partial_k g(k) \simeq \int_0^\infty \mathrm{d}b\; b\, e^{-\mathrm{Im}\,\chi_{\mathrm{C}}}\cos\big(\mathrm{Re}\,\chi_{\mathrm{C}}\big)\,\mathrm{Re}\,\partial_k\chi_{\mathrm{C}} < 0. \quad (4.10)$$

Therefore, from (3.20) together with (4.9) the time delay has a positive value. Note that delayed emission of a particle from the interaction region does not always correspond to late occurrence of scattering. The occurrence of collisions in our approach is concerned with the overlap (2.6) relevant to emerging wave packets.

5 Conclusion

The present wave packet formulation provides a method of defining and examining the occurrence of nuclear collisions. On the basis of the arising wave packet during the interaction the occurrence time proves to be presented in terms of the off-shell scattering matrix element, and is then employed to obtain the occurrence position using the probability density of a projectile. In order to give a typical and plain application of the method we have dealt with the occurrence of the scattering in the Dirac approach where narrow wave packets are available for analysis. The result reveals that at higher intermediate energies the forward scattering occurs before the peak of the nucleon probability density passes the center of the nuclear interaction region.

The present adoption of potential parameters leads to the reasonable result that the scattering takes place within the interaction region. The consequence $-R < \zeta < 0$ originates from the strong absorption and the function $e^{i(k-p)z}$ peculiar to the off-shell matrix element. The former not only restricts the region of contribution in the integral for the matrix element but also therein yields the dominant term in combination with the latter.

The position ζ depends on the packet width, and approaches the center of the interaction region with the decreasing width. Using the time-independent width, the packet may be taken so narrow that the position is found to be near the point corresponding to the peak of the free particle probability density at the occurrence time. On the whole because of the sufficiently high velocity the scattering in our consideration turns out to occur in the vicinity of the point independent of the packet size.

Appendix A. Off-shell matrix element

Computation of the off-shell matrix element is rather involved due to $p \neq k$, and proceeds as follows. As in [5] we use

$$W = \begin{pmatrix} (V+S)\sigma^0 & 0 \\ 0 & (V-S)\sigma^0 \end{pmatrix},$$

σ^0 being the 2×2 unit matrix. Then from (3.1) and (3.7) there results

$$\hat{T}(\mathbf{p},\mathbf{k}) = \int \mathrm{d}^3x\, e^{-i\mathbf{p}\cdot\mathbf{r}} \times\left[V+S-i\frac{V-S}{\{E(p)+m\}D}(\boldsymbol{\sigma}\cdot\mathbf{p})(\boldsymbol{\sigma}\cdot\nabla)\right]e^{i(\mathbf{k}\cdot\mathbf{r}+Q)}. \quad \text{(A.1)}$$

Integration by parts of the term with the gradient using $S = V = 0$ for $r \to \infty$ yields

$$\hat{T}(\mathbf{p},\mathbf{k}) = \int \mathrm{d}^3x \frac{e^{-i\mathbf{s}\cdot\mathbf{r}}}{D}\left[D(V+S) + \frac{p^2(V-S)}{E(p)+m} + i\frac{E(k)+m}{E(p)+m}\frac{(\boldsymbol{\sigma}\cdot\mathbf{p})\{\boldsymbol{\sigma}\cdot\nabla(V-S)\}}{D}\right]e^{iQ}. \quad \text{(A.2)}$$

From $(\boldsymbol{\sigma}\cdot\mathbf{A})(\boldsymbol{\sigma}\cdot\mathbf{B}) = \mathbf{A}\cdot\mathbf{B} + i\boldsymbol{\sigma}\cdot(\mathbf{A}\times\mathbf{B})$ with $\mathbf{A} = \mathbf{p}$ and $\mathbf{B} = \nabla(V-S)$, we have the expression (3.9) together with (3.10) and (3.11).

Let $\mathbf{b}$ be given as

$$\mathbf{b} = b\cos\varphi\mathbf{e}_1 + b\sin\varphi\mathbf{e}_2, \quad \text{(A.3)}$$

where $\mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{n}$ and $\mathbf{e}_2 = (\mathbf{k}\times\mathbf{p})/|\mathbf{k}\times\mathbf{p}|$. Further, we write $h = \chi^\dagger\boldsymbol{\sigma}\cdot\mathbf{n}\chi = \pm 1$, which for a vector $\mathbf{A}$ leads to

$$\chi^\dagger\boldsymbol{\sigma}\cdot\mathbf{A}\chi = h\mathbf{n}\cdot\mathbf{A}. \quad \text{(A.4)}$$

Using

$$\exp[iK\boldsymbol{\sigma}\cdot\mathbf{A}] = \cos(K|\mathbf{A}|) + i\frac{\boldsymbol{\sigma}\cdot\mathbf{A}}{|\mathbf{A}|}\sin(K|\mathbf{A}|), \quad \text{(A.5)}$$

where K is a complex scalar, it follows that:

$$\chi^\dagger\exp\left[iQ_1\boldsymbol{\sigma}\cdot(\mathbf{b}\times\mathbf{k})\right]\chi = \cos(kbQ_1), \quad \text{(A.6)}$$

$$\chi^\dagger\boldsymbol{\sigma}\cdot(\mathbf{r}\times\mathbf{p})\exp\left[iQ_1\boldsymbol{\sigma}\cdot(\mathbf{b}\times\mathbf{k})\right]\chi = -hpb\sin\theta\sin\varphi\cos(kbQ_1) +ip(b\cos\theta - z\sin\theta\, e^{ih\varphi})\sin(kbQ_1). \quad \text{(A.7)}$$

The expression (3.12) then results from $\mathbf{s}\cdot\mathbf{r} = pb\sin\theta\cos\varphi + z(p\cos\theta - k)$ and the Bessel functions of integer order

$$J_n(x) = \frac{1}{2\pi i^n}\int_0^{2\pi}\mathrm{d}\varphi\, e^{ix\cos\varphi}\cos(n\varphi). \quad \text{(A.8)}$$

According to the convention of the eikonal approximation the z-axis along the direction of the average momentum $(\mathbf{p}+\mathbf{k})/2$ is taken, and for the on-shell scattering amplitude the momentum transfer $\mathbf{p}-\mathbf{k}$ is thereby perpendicular to the z-axis. However, the orthogonality does not hold for the off-shell one because of $p \neq k$. Further, as $e^{i(k-p\cos\theta)z}$ appears in (3.12), the corresponding function peculiar to the off-shell matrix element arises also when we use the convention. In this paper the convention is not used since we consider the forward scattering.

Appendix B. Probability density

The probability density is expressed as

$$\rho = \int \frac{\mathrm{d}^3q'}{E(k'')}\frac{\mathrm{d}^3q}{E(k')}a(q')a(q)e^{i[E(k'')-E(k')](t-t_0)} \times\psi^{(+)\dagger}(\mathbf{k}'',\mathbf{r})\psi^{(+)}(\mathbf{k}',\mathbf{r}), \quad \text{(B.1)}$$

where $\mathbf{k}' = \mathbf{k}+\mathbf{q}$ and $\mathbf{k}'' = \mathbf{k}+\mathbf{q}'$. Here we write $Q' = Q(\mathbf{k}',\mathbf{r})$ and $Q'' = Q(\mathbf{k}'',\mathbf{r})$. As for $\psi^{(+)}(\mathbf{k}',\mathbf{r})$ given by replacing $\mathbf{k}$ in (3.1) with $\mathbf{k}'$ the $\mathbf{q}$-dependence of the phase factor alone is assumed to be crucial for the integration. Further, in the lower component we write $\boldsymbol{\sigma}\cdot\nabla e^{i(\mathbf{k}'\cdot\mathbf{r}+Q')} \simeq i\boldsymbol{\sigma}\cdot(\mathbf{k}'+\nabla Q')e^{i(\mathbf{k}'\cdot\mathbf{r}+Q')}$ and $\boldsymbol{\sigma}\cdot(\mathbf{k}'+\nabla Q') \simeq \boldsymbol{\sigma}\cdot\mathbf{k}$, the latter being due to $k \gg |\nabla Q_0|$, $|kb\nabla Q_1|$. The lower component of $\psi^{(+)}(\mathbf{k}'',\mathbf{r})$ is similarly handled. There appear $\mathbf{k}'\cdot\mathbf{r} - E(k')(t-t_0) + Q'_0$, and its counterpart depending on $\mathbf{k}''$ in the exponents. We expand these in powers of $\mathbf{q}$ and $\mathbf{q}'$, and keep the first two terms in each expansion.

Now the integrand has

$$\chi^\dagger\exp\left[-iQ_1''^{*}\boldsymbol{\sigma}\cdot(\mathbf{b}\times\mathbf{k}'')\right]\exp\left[iQ'_1\boldsymbol{\sigma}\cdot(\mathbf{b}\times\mathbf{k}')\right]\chi. \quad \text{(B.2)}$$

This is rewritten by (A.5). The factors other than the trigonometric functions vary comparatively slowly with respect to $\mathbf{q}$ and $\mathbf{q}'$. Then we may replace $\mathbf{k}'$ and $\mathbf{k}''$ by $\mathbf{k}$ in the factors. Hence (B.2) becomes $\cos(Q'_1|\mathbf{b}\times\mathbf{k}'| - Q_1''^{*}|\mathbf{b}\times\mathbf{k}''|)$ from $\chi^\dagger\boldsymbol{\sigma}\cdot(\mathbf{b}\times\mathbf{k})\chi = 0$ obtained by (A.4). The terms in the argument are both linearized with respect to $\mathbf{q}$ and $\mathbf{q}'$ so that $Q'_1|\mathbf{b}\times\mathbf{k}'| = kbQ_1 + \nabla_k(kbQ_1)\cdot\mathbf{q}$ and so forth. From (2.11) it follows that:

$$\rho = \frac{\Lambda}{2}\left[e^{-2\,\mathrm{Im}\,Q_+}\left|G(\mathbf{R}_+)\right|^2 + e^{-2\,\mathrm{Im}\,Q_-}\left|G(\mathbf{R}_-)\right|^2\right], \quad \text{(B.3)}$$

where $\Lambda = \{N(k)\}^2(1+k^2/|D(k,r)|^2)$ and

$$\mathbf{R}_\pm = \mathbf{r} - \mathbf{u}(t-t_0) + \nabla_k Q_\pm = \mathbf{b} + \left[z - u(t-t_0) + \partial_k Q_\pm\right]\mathbf{n}. \quad \text{(B.4)}$$

Bearing (2.14) in mind we obtain (3.25) together with (3.26).

The above evaluation is made in disregard of the time dependence of the packet width. Examination of the time dependence is to be performed in sect. 4 and Appendix C, and to warrant the use of the constant packet size for our analysis by means of values of the packet width and the nuclear radius.

Appendix C. The time dependence of the wave packet width

In order to employ narrow wave packets we here examine the time dependence of the width of the probability density arising due to the terms quadratic in $\mathbf{q}$. Neglecting the spin orbit interaction it follows from (B.1) that

$$\rho = \Lambda e^{-2\,\mathrm{Im}\,Q_0(k,z,b)}\left|\Gamma(\mathbf{k},\mathbf{r},t)\right|^2, \qquad \text{(C.1)}$$

where

$$\Gamma = \int \frac{\mathrm{d}^3 q}{E(k')} a(q) e^{iP} \approx \frac{A}{E(k)} \int \mathrm{d}^3 q\, e^{-d^2 q^2 + iP} \qquad \text{(C.2)}$$

with $P = \mathbf{q}\cdot[\mathbf{b} + (z + \partial_k Q_0)\mathbf{n}] - \Delta E(t - t_0)$, the second derivative of Q_0 being negligible. Expansion of $E(k')$ in powers of $\mathbf{q} = (q_1, q_2, q_3)$ yields

$$\Delta E = E(k') - E(k) = \frac{q_1^2 + q_2^2}{2E(k)} + u q_3 + \frac{m^2}{2E(k)^3} q_3^2 \qquad \text{(C.3)}$$

with $u = k/E(k)$. Then integration over each of the components of $\mathbf{q}$ in (C.2) leads to

$$\rho = \frac{I}{\sqrt{1+\mu}(1+\nu)} \exp\left[-2\,\mathrm{Im}\,Q_0 - \frac{1}{2d^2}\left\{ \frac{(z - u(t-t_0) + \mathrm{Re}\,\partial_k Q_0)^2}{1+\mu} + \frac{b^2}{1+\nu} \right\}\right], \qquad \text{(C.4)}$$

where

$$\mu = \left(\frac{m}{E(k)}\right)^4 \nu, \qquad \nu = \left[\frac{u(t-t_0)}{2kd^2}\right]^2. \qquad \text{(C.5)}$$

Hence μ and ν in the exponent govern the time dependence of the packet size. Obviously, with the passage of time the probability density distribution shrinks before $t = t_0$ and spreads after $t = t_0$. The result $\mu < \nu$ is a relativistic effect.

When $u|t - t_0|$ in (C.5) is replaced by R, we have $\mu = 1.7 \times 10^{-2}$ and $\nu = 2.0 \times 10^{-1}$, for example, at $T_p = 800\,\mathrm{MeV}$ in table 1. Accordingly, in spite of the relativistic effect and the small value $d = 1.0\,\mathrm{fm}$ the size of the initial wave packet the center of which is near the point $z = -R$ with $b = 0$ proves to be still sufficiently small compared with that of the target nucleus. Incidentally, the choice $d = 0.5\,\mathrm{fm}$ leads to a result $\nu = 3.2$. The value $d = 1.0\,\mathrm{fm}$ then suffices for the present analysis with respect to the determination of the occurrence position.

References

1. T. Suzuki, Ann. Phys. (N.Y.) **189**, 352 (1989).
2. H. Feshbach, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1992) and references therein.
3. T. Suzuki, I. Maeda, Z. Phys. A **347**, 75 (1993); T. Suzuki, in *Proceedings of the 14th International Conference on Particles and Nuclei*, edited by C.E. Carlson, J.J. Domingo (World Scientific, Singapore, 1997) p. 679.
4. N. Austern, *Direct Nuclear Reaction Theories* (Wiley, New York, 1970); M.L. Goldberger, K.M. Watson, *Collision Theory* (R.E. Krieger Publishing Co., New York, 1975).
5. R.D. Amado, J. Piekarewicz, D.A. Sparrow, J.A. McNeil, Phys. Rev. C **28**, 1663 (1983).
6. R.D. Amado, J. Piekarewicz, D.A. Sparrow, J.A. McNeil, Phys. Rev. C **29**, 936 (1984).